EXTREMAL METRICS ON SASAKIAN MANIFOLDS

MIHAELA VERONICA PILCA

Master Thesis
Şcoala Normală Superioară Bucureşti (SNSB), August 2006
Supervisior: Prof. Dr. Liviu Ornea

Abstract:

In my master thesis I have determined the analogue of extremal Kähler metrics on Sasakian manifolds and proved some of their properties.

The extremal Kähler metrics were introduced by Calabi as critical points of a Riemannian functional. His main motivation was to find canonical representants of a given Kähler class. These metrics are also a generalization of Kähler-Einstein metrics and of metrics with constant scalar curvature.

The aim of my master thesis was to determine on Sasakian manifolds, which are the analogue of Kähler manifolds in odd dimensions, a class of special metrics corresponding to extremal Kähler metrics and which, in their turn, generalize the class of Sasaki-Einstein metrics. I have showed that there are two equivalent different ways in which Sasakian extremal metrics may be introduced. A first possible approach is a "formal" one: I considered for contact manifolds the corresponding notion of holomorphic vector fields and asked then for an extremal Sasakian metric to have the scalar curvature a contact-holomorphic potential. This definition allowed us to establish the relationship with the extremal metrics on the two Kähler manifolds between which any Sasakian manifold naturally stays (the metric cone and the quotient manifold, in the regular case). The second approach is the variational one, which also justifies the name "extremal", where I considered the critical points of the Riemannian functional of Calabi. The main problem here was to establish the corresponding space of metrics on which this functional is defined. The main results of this work are the equivalence of these two approaches and the proof of a decomposition of the Lie algebra of contact-holomorphic vector fields on a compact extremal Sasakian manifold.